**To integrate x to power x**

There is no elementary function for finding the integral (at least to me). However, here is a challenge for you to find the definite integral .

**(a)** Given , express in terms of an infinite series.

**(b)** Use integration by parts, or otherwise, show that :

, where ln stands for the logarithm base e and we don’t write the integrating constant for simplicity.

**(c) (i)** Use l’Hospital’s rule to show that

**(ii)** Show that

And

**(d)** Prove that .

**(e)** Prove that .

**(a)**

Since , we have

**(b) Method 1**

Let . Then

Using integration by parts, we get:

**Method 2**

Integrate both sides,

Therefore,

**(c) (i)**

= 0

**(ii)** By **(b)**,

, by **(c)** **(i)**  where m = n + 1

, by **(b)** and **(c) (i)**

**(d)**  , by **(a)**

, by **(c) (ii)**

**(e) Check:**

**Yue Kwok Choy**

**23/1/2017**