**To integrate x to power x**

 There is no elementary function for finding the integral (at least to me). However, here is a challenge for you to find the definite integral .

**(a)** Given , express in terms of an infinite series.

**(b)** Use integration by parts, or otherwise, show that :

 , where ln stands for the logarithm base e and we don’t write the integrating constant for simplicity.

**(c) (i)** Use l’Hospital’s rule to show that

 **(ii)** Show that

 And

**(d)** Prove that .

**(e)** Prove that .

**(a)**

 Since , we have

**(b) Method 1**

 Let . Then

 Using integration by parts, we get:

 **Method 2**

 Integrate both sides,

 Therefore,

**(c) (i)**

 = 0

 **(ii)** By **(b)**,

 , by **(c)** **(i)**  where m = n + 1

 , by **(b)** and **(c) (i)**

**(d)**  , by **(a)**

 , by **(c) (ii)**

**(e) Check:**

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